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Associative Memory Models for Mental Processes: Connections with q-Statistical Mechanics^{*}

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Abstract

We present here a review of our modelling efforts in recent years based on associative memory, artificial neural networks, to illustrate the main basic mechanisms of neurotic mental behavior as described by Freud. We proposed, that neurotic behavior may be understood as an associative memory process in the brain, and that the symbolic associative process involved in psychoanalytic working-through can be mapped onto a corresponding process of reconfiguration of the neural network. The connection of symbolic processing to sensorial memory traces illustrates a phenomenological view of the mind, where consciousness is deeply rooted in sensorial experience with the environment and the association of symbols to meaning. These associative memory models for mental processes suggest that q-MaxEnt distributions may be relevant for the study of these neural models. We therefore also review our recent work regarding dynamical mechanisms leading to q-MaxEnt distributions in memory neural networks, when these are modeled by nonlinear Fokker-Planck equations.

Keyphrases

Consciousness, unconsciousness, mental processes, metarepresentations, self-organized associative-memory neural networks, entropic measures, generalized statistical mechanics, nonlinear Fokker-Planck equations.

Introduction

There has been much effort in recent decades to develop mathematical-computational models of mental phenomena and processes studied and treated by psychologists, psychiatrists and neuroscientists (Freud 1958; Freud 1966; Kandel et al. 2000; Kandel 2005), such as creativity, delusions, disorganized thought, schizophrenia and the neuroses (Cleeremans et al. 2007; Taylor and Villa 2001; Taylor 2011; Carvalho et al. 2003; Wedemann, Carvalho, and Donangelo 2006b; Wedemann, de Carvalho, et al. 2008; Wedemann, Donangelo, et al. 2009a). Psychodynamical theories correlate creativity, psychopathology and unconsciousness, and aspects such as broader, distant or looser associations and unfocusing of attention are common in describing such mental processes. Memory functioning is a crucial aspect in these char-

acterizations and it is thus highly important to approach the problem of describing the mechanisms whereby we remember. How do we reminisce both consciously and unconsciously? Artificial associative memory models (AMMs) have been widely used as artificial storage devices and also to represent an approximation of human memory functioning. Some examples of these models are the Hopfield and Cohen-Grossberg models (Hopfield 1984; Cohen and Grossberg 1983). We have used artificial neural networks, and in particular AMMs, to develop illustrative, schematic, self-organizing, neurocomputational models to describe mechanisms underlying mental processes, both normal and pathological, and to represent the interplay between conscious and unconscious mental activity (Carvalho et al. 2003; Wedemann, Carvalho, and Donangelo 2006a; Wedemann, Carvalho, and Donangelo 2006b; Wedemann, de Carvalho, et al. 2008; Wedemann, Donangelo, et al. 2009a; Wedemann, Donangelo, et al. 2009b; Wedemann, Carvalho, and Donangelo 2011; Siddiqui et al. 2018; Wedemann and Carvalho 2012).

Stochastic versions of AMMs, such as the Boltzmann machine (Hertz et al. 1991) and Generalized Simulated Annealing (GSA) (Tsallis and Stariolo 1996), involve entropic measures. Therefore, it is essential to understand how fundamental concepts, theories and methods from statistical mechanics can be used to support the development of models of associative memory (Tsallis 2023). Here we review our previous modeling efforts regarding associative memory, with a focus on neurotic mental functioning (Freud 1966; Kandel 2005). We will discuss the main features of the algorithms that we have developed for our model and how fundamental, statistical mechanical, theoretical tools are used in these developments.

Freud and the Neuroses

We have approached the challenge of developing a computational model with a neuronal mechanism to describe neurotic mental functioning by considering three very basic findings of Sigmund Freud (Freud 1958; Freud 1966). First, he detected in his work with neurotic patients that traumatic and repressed memories are knowledge which is present in their memories although they are incapable, momentarily or permanently, to represent them symbolically, *i.e.*, these memories are inaccessible to the patient's consciousness, and he thus called it *unconscious knowledge*. His second very important observation is that these patients systematically repeated symptoms in the form of ideas and impulses, a tendency that he called a *compulsion to repeat*. He was able to relate this compulsion to repeat to the traumatic and re-

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pressed memories of the patient, and this causal relationship is one of his profoundly significant contributions. In a next third seminal step, he found that neurotic patients have obtained relief and cure of their painful symptoms through a mechanism that he called *working-through*. This technique aims at having the patient access unconscious memories in order to elaborate knowledge regarding the causes of symptoms in order to understand and change his compulsion to repeat. It is a process of symbolization that consists of analyzing symptoms, dreams, free associative talking, parapraxes (forgetting, slips of the tongue and pen, misreading, etc.), and also that which is acted out by the patient in transference between him and the psychoanalyst. By working-through in psychoanalytic sessions, the patient represents through symbolical mechanisms such as language his originally unconscious memories.

A Computational Model for the Neuroses

We proposed a computational model where the neuroses are described by an associative memory process (Wedemann, de Carvalho, et al. 2008; Wedemann, Donangelo, et al. 2009a; Siddiqui et al. 2018). An associative memory is a mechanism whereby a network, when presented with an input pattern S, accesses a specific stored pattern S' that is most similar to S than the other stored patterns. The compulsion to repeat a neurotic symptom or act is then described as a response of the network to a stimulus S which resembles a repressed (unconscious) memory trace S'. Stimulus S causes the neural network to stabilize on S', corresponding to a minimal energy state of the network, that activates the neurotic act. Thus, in neurotic behavior, the symptom is not a response to a stimulus as a novel experience, but is a consequence of triggering the access to a repressed memory S'.

Our model consists of a memory structure with two neural network modules representing sensorial and symbolic memories (see Fig. 1, Wedemann, Donangelo, et al. (2009a)). Sensorial memory stores traces that represent mental images of stimuli received by sensory receptors of the body, including information regarding affects and emotion. It represents areas of the brain that synthesize auditory, visual, and somatic information such as the cerebellum, reflex pathways, hippocampus, amygdala, and prefrontal, limbic, and parieto-occipital-temporal cortices. Symbolic memory stores higher-level representations of traces in sensorial memory, i.e., symbols. It represents structures in the brain such as areas of the hippocampus, the medial temporal lobe, Wernicke's and Broca's areas, and other areas of the frontal cortex. These areas of the brain are associated with the capacity to symbolize, such as with language, and they allow us to associate, for example, a verbal description, or maybe a melody or a painting with the visual sensation of seeing an object or a view. The two memory modules are connected and they interact, producing conscious and unconscious mental functioning. Stimuli from sensory receptors activate the retrieval of a pattern in sensorial memory that may become conscious, if it can trigger the access of a trace in symbolic memory. If the retrieval of a memory trace in sensorial memory can generate activity to access a pattern in symbolic memory, it can become conscious.

A stimulus that generates access to a trace in sensorial memory that does not, as a consequence, activate an access to a higher-order representation in symbolic memory remains unconscious. This is in agreement with Freud's discovery that repressed memories are those that cannot be represented symbolically, thus enforcing the importance of language in psychoanalytic treatment, and elucidating the fact that neurotics cannot explain the causes of their neurotic acts. The compulsion to repeat by neurotics (Freud 1958; Freud 1966) is thus explained

in our model as a bodily response (an act) to a retrieval in sensorial memory, which does not generate activity in symbolic memory, as that which occurs in a reflex. The incapacity to symbolize, i.e. to access and generate higher-order representations (or meta-representations) is represented in our model by weaker synaptic connections between the sensorial and symbolic memory modules. The psychoanalytical process of working-through then consists of reconfiguring and strengthening these inter-module connections. The relation of conscious and unconscious mental functioning to the capacity to generate higherorder representations has been described in the literature as higher order thought theory (Cleeremans et al. 2007; Wedemann and Carvalho 2012). Our computational model consists of three basic algorithms: a hierarchical clustering algorithm, a memory access mechanism, and a working-through algorithm. The detailed description of these component algorithms can be found in (Wedemann, Donangelo, et al. 2009a; Siddiqui et al. 2018). We review here some of their basic and physically relevant properties.

The hierarchical clustering algorithm is responsible for generating the topological structure of each of the two neural network memory modules. For the elaboration of this clustering algorithm, we considered basic microscopic biological mechanisms, such as the on-center/off-surround structure found in brain cells in many animals whereby a neuron cooperates with other neurons in its immediate neighborhood through excitatory synapses, whereas it competes with neurons that are located further outside these surroundings (see Wedemann, Donangelo, et al. (2009a) and references therein). Other basic mechanisms, such as the fact that synaptic strengthening among neurons is promoted by the simultaneous stimulation of the pair, as in Hebbian learning, have also served as inspiration.

This self-organizing algorithm (Wedemann, Donangelo, et al. 2009a) generates neuronal clusters, where a group of spatially close neurons have a higher probability of being adjacent in the network's graph, with stronger synapses among pairs of these neurons. This also represents a kind of preferential attachment mechanism with some conservation of energy (neurosubstances) among neurons, controlling synaptic plasticity and the formation of neuronal biological circuits also called maps. The algorithm also simulates the storage of external stimuli received by the network. As these competition-cooperation mechanisms are also controlled by the environment, they constitute the way that the environment represents itself in the brain (Wedemann, Carvalho, and Donangelo 2006a; Wedemann, Carvalho, and Donangelo 2006b; Wedemann, de Carvalho, et al. 2008; Wedemann, Donangelo, et al. 2009a).

One of the main findings of our completely self-organizing hierarchical clustering algorithm is that the emergent structure of the networks generated by this algorithm presents node degree distributions that have asymptotic, power-law-like forms. These distributions deviate significantly from Poisson distributions that would be present if the connections were distributed in a random way and, besides indicating structured organization of the network's topology, they also suggest that the study of these networks may benefit significantly from a description involving a nonextensive statistical mechanics, theoretical framework (Tsallis 2023).

The memory access mechanism is based on the stochastic generalization of the discrete Hopfield neural network model, called the Boltzmann machine (Hertz et al. 1991) that is based on the Sherrington–Kirkpatrick spin glass model. The network has N nodes (neurons) where each node i has a discrete state $S_i \in \{-1,1\}$ and functions as

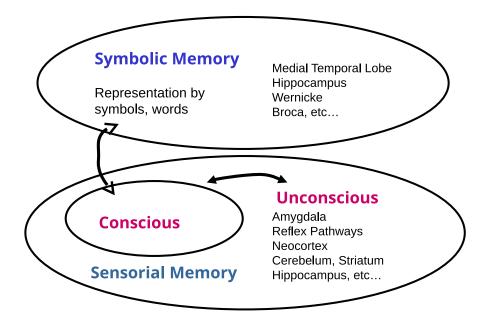


Figure 1: Memory structure with two modules, where one stores sensory input and the other stores symbolic representations. In sensorial memory there are traces that can or cannot become conscious.

a McCulloch-Pitts neuron, so that its evolution equation is

$$S_i(t+1) := extstyle \operatorname{\mathsf{sgn}}\left(\sum_j \omega_{ij} S_j(t) - heta_i
ight),$$
 (1)

where θ_i is the firing threshold and

$$\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x \ge 0; \\ -1, & \text{if } x < 0. \end{cases}$$
 (2)

The synaptic weights $w_{ij}=w_{ji}$ that connect node i and j are symmetric.

As a consequence of the symmetry of the weights, it is possible to define an energy function, representing the potential energy corresponding to the interactions between neurons,

$$E(\{S_i\}) = -\frac{1}{2} \sum_{ij} w_{ij} S_i S_j.$$
 (3)

Stored memories then correspond to the minimum energy (stable) states, which are attractors of the dynamics (1) in the memory retrieval mechanism. The dynamics of the network follows a trajectory that always goes downhill on the energy surface. Memory retrieval is then achieved in the Boltzmann machine with a simulated annealing process. The energy surface is sampled according to the Boltzmann-Gibbs transition probability from state ${\bf S}$ to ${\bf S}'$ which, for a temperature T, if $E({\bf S}') > E({\bf S})$, is given by

$$P_{BG}(m{S} o m{S}') = \exp \left[rac{-(E(m{S}') - E(m{S}))}{T}
ight] \,.$$
 (4)

When E(S') < E(S), transitions are always accepted.

Following the suggestion that we mentioned previously, that a description involving a nonextensive statistical mechanics may be advantageous, we also implemented memory retrieval using Generalized

Simulated Annealing (GSA, Tsallis and Stariolo (1996)). In this case, the transition probability, for a value of an entropic parameter q, is given by

$$P_{GSA}(S \to S') = \frac{1}{\left[1 + (q-1)\frac{(E(S') - E(S))}{T}\right]^{\frac{1}{q-1}}}$$
 (5)

Thermostatistics and Memory Networks

The application of the maximum entropy principle to generalized, non-standard entropic functionals is useful for the study of many complex systems, including those investigated in the neurosciences and artificial intelligence (AI). These systems often exhibit probability distributions or densities that differ from the standard, exponential, Boltzmann-Gibbs form. The generalized maximum S_q entropy scheme generates distributions that are consistent with power-law distributions frequently observed in neuroscience, both in experimental research and in theoretical numerical studies (see Wedemann, Donangelo, et al. (2009a); Siddiqui et al. (2018) and references therein).

For a probability density $\mathcal{P}(\boldsymbol{r})$, the power-law entropic functional S_q is

$$S_q[\mathcal{P}] = \frac{1}{1-q} \int \mathcal{P} \left[\left(\frac{\mathcal{P}}{\mathcal{P}_c} \right)^{q-1} - 1 \right] d^M \boldsymbol{r},$$
 (6)

where q is a real parameter called the entropic index, $r \in \mathbb{R}^M$, and \mathcal{P}_c is a constant with the same dimensions as $\mathcal{P}(r)$. In the limit $q \to 1$, the entropy S_q becomes the standard Boltzmann-Gibbs (BG) entropy,

$$S_{BG} = S_1 = -\int \mathcal{P} \ln(\mathcal{P}/\mathcal{P}_c) d^M \boldsymbol{r}.$$
 (7)

The generalized thermostatistical formalism associated with the entropic functional (6) is based on the optimization of (6), restricted by the constraints of normalization and the mean value of an energy function

 $\varepsilon(\boldsymbol{r})$,

$$\int \mathcal{P}(m{r}) \, d^M m{r} = 1$$
, and $\int \mathcal{P}(m{r}) \, arepsilon(m{r}) \, d^M m{r} = \mathcal{E}$. (8)

The q-exponential canonical distribution (q-MaxEnt distribution),

$$\exp_q\left(-\frac{\mathcal{E}}{T}\right) = \begin{cases} \left[1-(1-q)\frac{\mathcal{E}}{T}\right]^{\frac{1}{1-q}} \;, & \text{for } 1-(1-q)\frac{\mathcal{E}}{T}>0 \;, \\ 0 \;, & \text{for } 1-(1-q)\frac{\mathcal{E}}{T}\leq 0 \;, \end{cases} \tag{9}$$

results from the constrained optimization of the S_q entropy (6). For q >1, these q-exponential canonical distributions behave asymptotically as power-laws and various complex systems that exhibit power-law behavior are actually described by them.

There are indications that symbolic structures present in the mind, such as language, may reveal correlations that follow distributions with a power-law behavior. For example, Zipf's law for the frequency of words (and communication signals in general) suggests that power-law correlations occur in human language, and even in animal communication. Since we have found power laws in the quantities that characterize the topologies generated by our clustering algorithm and in experimental data related to our studies (Wedemann, Donangelo, et al. 2009a; Siddigui et al. 2018), we have used GSA with the transition probability (5) to implement our memory retrieval mechanism.

In the simulation experiments that we have conducted with our model (Wedemann, Donangelo, et al. 2009a; Siddiqui et al. 2018; Wedemann, Carvalho, and Donangelo 2011), we found that when memory is hierarchically structured, with both short and long-range synapses, and memory functioning is processed with GSA, the neuronal system should then be capable of making more distant associations among stored memory traces, with more metaphors and creativity, than a system governed by Boltzmann-Gibbs statistics. Values of the temperature parameter T and entropic parameter q regulate these capabilities, reflecting properties of the (biological) network, such as the availability of neuromodulators and neurotransmitters. With GSA we were also able to obtain a power-law-like asymptotic behavior for the propagation of signals in the network that agree with experimental results showing power laws for the propagation of signals in the brain (Siddiqui et al. 2018).

These findings have lead us to engage in the investigation of possible dynamical mechanisms that can generate generalized maximum entropy distributions in memory neural networks (Wedemann and Plastino 2016; Wedemann, Plastino, and Tsallis 2016; Wedemann and Plastino 2017: Luca et al. 2018: Wedemann and Plastino 2019: Wedemann and Plastino 2020; Wedemann and Plastino 2021; Wedemann and Plastino 2023; Wedemann, Plastino, Tsallis, and Curado 2024; Luca et al. 2025; Wedemann and Plastino 2026). We enumerate some of these efforts in the following section.

The Fokker-Planck Formalism

It is possible to generalize the McCulloch-Pitts, discrete activation neural model so that the output signal of a neuron i (in equilibrium) is given by a continuous function. The Cohen-Grossberg general model (Cohen and Grossberg 1983) describes a wide family of dynamical systems that are composed of co-evolving, interacting elements (neurons) with continuous state variables, and a particular instance is the continuous Hopfield, associative memory, neural network model (Hopfield 1984).

When studying complex dynamical systems with many elements, each described by a continuous state variable, such as a neural network with N neurons where each one has a continuous activation represented by x_i , instead of following the evolution of a single instance of the system, it is often more feasible to consider the time evolution of a statistical ensemble of identical copies of the system with different initial conditions. This ensemble can then be described by $\exp_q\left(-\frac{\mathcal{E}}{T}\right) = \begin{cases} \left[1-(1-q)\frac{\mathcal{E}}{T}\right]^{\frac{1}{1-q}} \;, & \text{for } 1-(1-q)\frac{\mathcal{E}}{T}>0 \;, \\ 0 \;, & \text{for } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \end{cases} \quad \text{for } 1-(1-q)\frac{\mathcal{E}}{T}>0 \;, \\ \text{for } 1-(1-q)\frac{\mathcal{E}}{T}>0 \;, \end{cases} \quad \text{a time-dependent probability density in N-dimensional phase space for } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves according to } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \text{ that, in the presence of noise, evolves } 1-(1-q)\frac{\mathcal{E}}{T}<0 \;, \quad \mathcal{P}(x_1,\ldots,x_N,t) \;, \quad \mathcal{P}(x_1,\ldots,$ a Fokker-Planck equation (FPE)

$$\frac{\partial \mathcal{P}}{\partial t} = D\nabla^2 \mathcal{P} - \boldsymbol{\nabla} \cdot [\boldsymbol{K}\mathcal{P}], \qquad (10)$$

where $\mathcal{P} = \mathcal{P}({m x},t)$, ${m x} \in \mathbb{R}^N$, ${m K} \in \mathbb{R}^N$ is called the drift field and D is a constant diffusion coefficient (see Wedemann, Plastino, and Tsallis (2016) and references therein).

Physical systems characterized by features such as inhomogeneity, spatial disorder, long-range interactions, overdamped dynamics and certain asymmetries may require a more general setting than the FPE, given by the nonlinear Fokker-Planck equation (NLFPE, Ribeiro et al. (2011)). Indeed complex systems with such characteristics frequently exhibit q-exponential canonical (q-MaxEnt) distributions. The complex system community has thus, in recent years, conducted intense research efforts into investigating diverse aspects of evolution equations including nonlinear difusion terms, such as the NLFPEs, and their applications to various problems in physics, biology and other fields (Ribeiro et al. 2011; Wedemann, Plastino, and Tsallis 2016; Luca et al. 2025; Lucchi et al. 2026).

We have thus used the nonlinear Fokker-Planck equation (NLFPE) with the form

$$\frac{\partial \mathcal{P}}{\partial t} = D\nabla^2[\mathcal{P}^{2-q}] - \boldsymbol{\nabla} \cdot [\boldsymbol{K}\mathcal{P}], \qquad (11)$$

and also some other variant, similar forms to study artificial memory neural networks which may deviate from the linear description. As we are interested in retrieving stored memory states of these networks, we search for possible stationary solutions to Eq.(11), which correspond to attractor states of the network's dynamics. We have found in our models that stationary states of NLFPEs describing our model networks have the form of q-exponential canonical distributions (Wedemann and Plastino 2016; Wedemann, Plastino, and Tsallis 2016; Wedemann and Plastino 2017; Luca et al. 2018; Wedemann and Plastino 2019; Wedemann and Plastino 2020; Wedemann and Plastino 2021; Wedemann and Plastino 2023; Luca et al. 2025; Wedemann and Plastino 2026).

As a notable example of these modeling efforts with the NLFPE, we have treated asymmetries in synaptic connections by considering a drift term in the NLFPE that does not arise from the gradient of a potential V, $K \neq -\nabla V$, and therefore K is a curl force (Wedemann and Plastino 2016; Wedemann, Plastino, and Tsallis 2016; Luca et al. 2018; Luca et al. 2025). In this case, the numerical solution for an instance of the NLFPE for two interconnected neurons shows that the activation functions of the two neurons spiral into limit cycles so that, in a stationary equilibrium condition, the attractor states correspond to a situation where the activation values of the two neurons rotate in the phase-space plane (x_1, x_2) , following an elliptical orbit (Luca et al. 2018; Luca et al. 2025). The emergence of these limit cycles indicates an important expression of the new types of dynamics that may arise from the asymmetric interactions.

Conclusion

Our schematic model for the neuroses illustrates how the ideas of Freud regarding the unconscious mind indicate that language, symbolic representations and meaning are essential for the emergence of conscious experience. In neurotic behavior, the subject is strongly attracted, by an associative memory mechanism (as in an attentional mechanism), to repressed or traumatic memories that determine his acts, of which he has no consciousness. In order to be conscious of an attended stimulus, a subject should be capable of reporting, by creating and associating meta-representations (abstract symbols) to sensory information.

Power-law-like distributions are observed in neuroscience, both in numerical simulations and in experimental observations. We have modeled continuous neural networks for associative memory using NLFP equations that present stationary solutions of the q-MaxEnt form. These q-exponential canonical distributions have asymptotic power-law-like behavior that correspond to attractor states in neuronal circuits and should account for the greater capacity to associate of the networks based on generalized statistical mechanics, when compared to those based on standard Boltzmann-Gibbs statistics.

We hope that this review will stimulate the curiosity of other researchers who may contribute to the many open issues related to these topics that still need to be explored. In this regard, any further developments will be very welcome.

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Contributions

All authors contributed equally to the paper.

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