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# The Logics Behind Reliable and Trustworthy Neuro-Symbolic Artificial Intelligence*

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## Book Review

Neuro-symbolic artificial intelligence (Sarker et al. 2021; Zhang and Sheng 2024), neurocompositional computing (Smolensky 1990; Smolensky et al. 2022), hyperdimensional computation (Plate 1995; Kanerva 2009), and vector-symbolic architectures (Gayler 2006; Graben, Huber, et al. 2022) — all referring to essentially the same kind of applied mathematics — play an increasing role for the development of explainable and trustworthy artificial intelligence (Samek et al. 2019) and for the design of non-conventional computing paradigms, such as neuromorphic or quantum computers (Melnikov et al. 2023).

Eduardo Mizraji, one of the pioneers of these intriguing research fields, has submitted *The Matrix Algebra of Logic* (Mizraji 2026), reviewing almost 40 years of intensive investigations on vectorial representations of symbolic logics (e.g. Mizraji (1989); Mizraji (1992); Mizraji (2008); Mizraji (2020)). Starting with an arbitrary assignment of binary (and later trinary) symbolic truth values to orthogonal (and later linear independent) vectors from an arithmetic vector space (called *filler vectors* in the related approach by Smolensky (1990)), logical connectives, such as negation, disjunction, or conjunction, become represented as linear operators, i.e. as *matrices*, upon the basic spaces. Introducing contextual information through vectorial tensor products (also known as *binding* in vector-symbolic architectures (Smolensky 1990; Graben and Potthast 2009)), allowed for a straightforward resolution of the pertinent XOR-problem of early neuronal connectionism (Minsky and Papert 1969). Moreover, moving from plain vector spaces to Hilbert spaces, where projections can be calculated, entails a probabilistic interpretation as required for quantum computation.

Reading Prof. Mizraji's new book was an inspiring experience where I got a lot of new insights about the connections between classical propositional logics with modal logics, the logics of counterfactuals by means of the "square root of negation," probabilistic logics, logical differential and integral calculus, and eventually the logics of cellular automata, such as Conway's famous "game of life."

More specifically, I have to mention Mizraji's interesting notes on Boole's original formalism, motivating his subsequent exposition of logical differential and integral calculus, in sect. 1.3; the emergence of incompatible Pauli spin matrices under vectorial basis transformations in sect. 4.1 (although not explicitly stated by the author); the introduction of a vector logical normal form in terms of "Kronecker polynomials" in sect. 4.5; and his recursive definition of the modal operators "necessity" and "possibility", inviting further research in terms of recurrent dynamical neural networks, in sect. 6.1.

I also see much potential for future investigations, not yet foreseen by the author. Writing the first Law of DeMorgan (p. 42) as

$$C = ND(N \otimes N)$$

(with $C$ as the conjunction matrix, $N$ as negation, and $D$ as disjunction), calls for an immediate representation of *The Matrix Algebra of Logic* in terms of Hopf algebras and quantum groups (Drinfel'd (1988), compare Graben and Gerth (2012); Sadrzadeh et al. (2013); Marcolli et al. (2023) for some related approaches). Introducing a matrix algebra co-product $\Delta(N) = N \otimes N$, would lead to DeMorgan's law

$$C = ND\Delta(N) \; ,$$

where all matrices are taken from the same domain now. Furthermore, it would be interesting to have not only a recursive modal vector logics, but also, e.g, another one for deontic logic with obligation and permission operators (McNamara and Putte 2025).

To conclude, Prof. Mizraji has submitted a highly inspiring and informative textbook on different fields of vector logics and vector-symbolic architectures that deserves to become a canonical standard work for those fields of scientific scholarship.

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